

# Estimating $\Omega$ from Galaxy Redshifts: Linear Flow Distortions and Nonlinear Clustering

B. C. BROMLEY<sup>1,2</sup>, M. S. WARREN<sup>1</sup>, AND W. H. ZUREK<sup>1</sup>

<sup>1</sup>Theoretical Astrophysics, MS B288, Los Alamos National Laboratory, Los Alamos, NM 87545

<sup>2</sup>Theoretical Astrophysics, MS-51, Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

## ABSTRACT

We propose a method to determine the cosmic mass density  $\Omega$  from redshift-space distortions induced by large-scale flows in the presence of nonlinear clustering. Nonlinear structures in redshift space such as fingers of God can contaminate distortions from linear flows on scales as large as several times the small-scale pairwise velocity dispersion  $\sigma_v$ . Following Peacock & Dodds (1994), we work in the Fourier domain and propose a model to describe the anisotropy in the redshift-space power spectrum; tests with high-resolution numerical data demonstrate that the model is robust for both mass and biased galaxy halos on translinear scales and above. On the basis of this model, we propose an estimator of the linear growth parameter  $\beta = \Omega^{0.6}/b$ , where  $b$  measures bias, derived from sampling functions which are tuned to eliminate distortions from nonlinear clustering. The measure is tested on the numerical data and found to recover the true value of  $\beta$  to within  $\sim 10\%$ . An analysis of IRAS 1.2Jy galaxies yields  $\beta = 0.8_{-0.3}^{+0.4}$  at a scale of 1,000 km/s which is close to optimal given the shot noise and the finite survey volume. This measurement is consistent with dynamical estimates of  $\beta$  derived from both real-space and redshift-space information. The importance of the method presented here is that nonlinear clustering effects are removed to enable linear correlation anisotropy measurements on scales approaching the translinear regime. We discuss implications for analyses of forthcoming optical redshift surveys in which the dispersion is more than a factor of two greater than in the IRAS data.

*Subject headings:* cosmology: theory — large-scale structure of universe — galaxies: clustering

## 1 Introduction

A redshift-space map of galaxies is distorted relative to the real-space galaxy distribution as a result of peculiar motions along an observer's line of sight. These distortions generate an anisotropy in pairwise correlations which would not appear in the real-space galaxy distribution of a statistically homogeneous and isotropic universe. Kaiser (1987) quantified the correlation anisotropy that results from large-scale peculiar flows in terms of the power spectrum of galaxies using the linear theory of gravitational instability. He demonstrated that in the linear regime the power measured by a distant observer only depends on the angle between the wavevector and the observer's line of sight, and on the dimensionless factor  $\beta \equiv \Omega^{0.6}/b$ . Here,  $\Omega$  is the cosmic mass density parameter and  $b$  is a function that differs from unity if the galaxies are a biased sample of the total mass. Hamilton (1992) transformed Kaiser's result out of the Fourier domain to determine the redshift-space correlation function  $\xi_s$  in linear theory and proposed an estimator of  $\beta$  based on a spherical harmonic decomposition of  $\xi_s$ . Subsequently, a number of  $\beta$  measurements from linear flow distortions have been reported (e.g., Hamilton 1993, 1995; Bromley 1994; Fisher, Scharf & Lahav 1993; Fisher et al. 1994; and Nusser & Davis 1994).

There are two major challenges for a statistical measurement of  $\beta$  from linear flows in a real galaxy redshift catalog. The first comes from the finite size of the catalog. The catalog must contain a fair sample of structure on a particular scale or else the correlation information will contain finite-sampling noise. A reasonable criterion is that sampling scales should be below  $\sim 10\%$  of the characteristic size of survey volume. The second challenge is to ensure that the sampling scales are large enough so that signal from nonlinear clustering does not contaminate the linear fluctuation modes. Fingers of God can extend up to several thousand kilometers per second in optically selected galaxy surveys and clearly affect the redshift-space power on these scales. (Here redshift-space distances are usually given in terms km/s; where real-space lengths are needed, we use the conventional units of  $h^{-1}$  Mpc where  $h$  is the Hubble parameter in

units of 100 km/s.) Thus the signal in redshift space from linearly growing fluctuation modes on scales from  $\sim 1,000$  km/s (characteristic of the translinear regime below which linear theory breaks down) to  $\sim 4,000$  km/s can be contaminated by strongly nonlinear features. Present-day catalogs are typically within a few times 10,000 km/s in radial extent, leaving at best only a small range of scales on which purely linear modes can be measured with a high degree of statistical integrity.

In this paper we address this problem of determining  $\beta$  from linear flows in the presence of nonlinear clustering. Our approach is motivated by a remarkable result from Peacock & Dodds (1994) concerning the redshift-space power spectrum  $P_s$ . While  $\xi_s$  itself can be modeled given the distribution of pairwise velocities, there is evidence (Fisher et al. 1994; Warren 1995) that this distribution has complicated behavior on a critical range scales from  $\sim 1\text{--}10 h^{-1}$  Mpc. In contrast, Peacock & Dodds (1994) found that nonlinear signal can be modeled simply and accurately in the Fourier domain on translinear scales and above. Specifically, the effects nonlinear clustering and linear flows in redshift space can be described by separable linear filters acting on the real-space power spectrum. The linear filtering hypothesis has been supported by tests with  $N$ -body simulations, although only mass particles and not collapsed galaxy halos were considered (Gramann, Cen & Bahcall 1993; Tadros & Efstathiou 1996). Here, we first suggest a form of the filters, which amounts to constructing a model for the anisotropy in  $P_s$ , and determine its validity in high-resolution numerical simulations using both mass particles and collapsed halos which are identified as galaxies. On the basis of this model we then suggest a method to extract  $\beta$  on scales of 1,000 km/s and above, even when the velocity dispersion is of comparable size. Finally, we apply our method to the IRAS 1.2Jy survey. The result is a new measure of  $\beta$  on translinear scales.

## 2 Power in Redshift Space

Following Peacock & Dodds (1994) we assume that the effects of line-of-sight peculiar motion on a galaxy distribution can be modeled by a linear filter function  $F$  that relates the real-space and redshift-space power spectra:

$$P_s(k, \mu) = F(k, \mu)P_r(k) , \quad (1)$$

where  $\mu$  is the angle cosine of the observer's line of sight and a wavevector of amplitude  $k$ . For definiteness we assume that the power is measured in a distant compact survey so that  $\mu$  is constant for all points in the survey.

The form of  $F$  can be motivated from its limiting behavior at small and large scales. At small scales, the peculiar motions are dominated by isotropic dispersion, and the redshift-space correlation function obtains from a simple convolution of the real-space function  $\xi_r$  and the distribution function  $f_v$  of pairwise radial velocities. According to evidence from simulations (Zurek et al. 1994) and observations (Marzke et al. 1995),  $f_v$  is accurately described by an isotropic exponential function at small pair separation,

$$f_v \sim \exp(-\sqrt{2}|v|/\sigma_v) , \quad (2)$$

where  $\sigma_v$  is the pairwise radial velocity dispersion. In the Fourier domain, the redshift-space power spectrum is thus related to the real-space power spectrum by a linear filter,  $F_D$ , the Fourier transform of  $f_v$  taken as a function of line-of-sight pairwise velocity:

$$F_D(k, \mu) = \left(1 + \frac{\mu^2 k^2 \sigma_v^2}{2}\right)^{-1} . \quad (3)$$

Turning to the large-scale regime, Kaiser's (1987) formula for a filter  $F_L$  which accounts for the anisotropy in redshift-space power from linear flows is

$$F_L = (1 + \beta\mu^2)^2 . \quad (4)$$

Peacock & Dodds (1994) found that an effective model for the desired filter  $F$  is obtained from a product of  $F_D$  and  $F_L$ . Following their ansatz we write equation (1) as

$$P_s(k, \mu) = \frac{(1 + \beta\mu^2)^2}{1 + \sigma_v^2 \mu^2 k^2 / 2} P_r(k) . \quad (5)$$

In committing to this model (eq. [5]) we limit our analysis to scales above the translinear regime. There is evidence (Brainerd et al. 1996) that Kaiser’s formula (eq. [3]) for the anisotropy in the redshift-space power spectrum breaks down at translinear scales. This breakdown can be quantified by including a “softening parameter” of roughly translinear scale into equation (3) which drives the filter to unity in the strongly nonlinear regime (see equation [5] in Brainerd et al. 1996). Theoretical insight into this effect is given by Fisher & Nusser (1996).

The model in equation (5) also may be limited by the assumption that the small-scale velocities are random and isotropic and exhibit none of the coherent flows that are generated by the linear or quasilinear dynamics. Numerical simulations support this assumption on megaparsec scales and below. A naive linear theory extrapolation of the pairwise velocity dispersion by way of the cosmological pair conservation equation (Peebles 1980 §71) also suggests that the linear flow contribution to  $\sigma_v$  at  $1 h^{-1}$  Mpc is small for most plausible cosmological scenarios. In the cold dark matter model discussed below, linear flows would contribute (in quadrature) only  $\sim 100$  km/s to the total of  $\sigma_v \approx 1,100$ , which amounts to less than a percent in  $\sigma_v^2$ . However the situation is more complicated on translinear scales where both random dispersion and significant coherent flows exist. Fortunately, the random motion contribution to the anisotropy of redshift-space fluctuation modes above the translinear regime comes primarily from strongly nonlinear scales where the velocity dispersion and spatial pairwise correlations are strongest and where the velocity distribution function takes a particularly simple form.

We note that the above expression differs from the result of Peacock & Dodds in the choice of the model for the small-scale velocity distribution function. Their work was based on a Gaussian model, however, as they and others (Cole, Fisher & Weinberg 1995; Brainerd et al. 1996) have noted, the difference between a Gaussian distribution and the exponential distribution is of little consequence in the quantification of redshift-space distortions. Nonetheless we advocate the exponential model because of its successes in describing both real and simulated data.

To determine the efficacy of the parameterization in equation (5) we examine the redshift-space power in numerical simulations. We ran two 17 million particle high-resolution N-body simulations in periodic cubes of size  $125 h^{-1}$  Mpc and  $500 h^{-1}$  Mpc, respectively, using a treecode with force smoothing of  $0.01 h^{-1}$  Mpc. In both cases the primordial power spectrum was based on cold dark matter (CDM) normalized to  $\sigma_8 = 0.74$  in an  $\Omega = 1$ ,  $h = 0.5$  universe. This cosmogony provides a reasonable match to the observed spatial clustering and pairwise velocity statistics of optically selected galaxies (Zurek et al. 1994; Marzke et al. 1995; Brainerd et al. 1996). We use the large-volume simulation to study the redshift distribution of mass on scales greater than  $50 h^{-1}$  Mpc; the small-volume run has sufficient mass resolution to identify halos as realistic galaxy candidates, hence it is used to study both the mass and galaxy distributions on smaller scales.

The results of the N-body analysis are presented in Figure 1, showing the anisotropy in redshift-space power spectrum of the CDM mass (Fig. 1a) and the full catalog of halos (Fig. 1b). We plot the power at constant  $k$  values as a function of the angle between the wavevector and the observer. Using the megaparsec separation value of  $\sigma_v$ , with values of 1,100 km/s and 850 km/s for mass and halos respectively, we find that the parameterization in equation (5) is successful on scales near and above the translinear regime. The breakdown in the model occurs on scales which are below  $\sigma_v$ , corresponding roughly to the characteristic size of the Fingers of God.

Remarkable is the extent to which the dispersion signal affects the angular dependence of the redshift-space power. In the case of the CDM mass distribution, there is negligible anisotropy at  $2\pi/k = 3,300$  km/s, three times  $\sigma_v$ , because the large-scale enhancement in power along the line-of-sight is almost completely canceled by small-scale dispersion.

Figure 1c depicts results from a third dataset, a cool subset of halos obtained by eliminating all objects which are within 3 Abell radii of the highest density peaks and which have peculiar velocities greater than 750 km/s. Although produced in a rather contrived manner, this culled dataset is suggestive of the IRAS 1.2Jy galaxies in that they both have low velocity dispersions compared with optical galaxies and have relatively small numbers of objects in the cores of rich optical clusters. Indeed, the resulting catalog has  $\sigma_v$  of 384 km/s and spatial clustering properties below  $100 h^{-1}$  Mpc which are consistent with the IRAS 1.2Jy data (Brainerd et al. 1996). Because of an antibias in the culled halo distribution relative to mass,  $\beta$  is about  $\sim 10\%$  above unity on translinear to linear scales with weak scale dependence. Given the great difference in small-scale clustering statistics between this dataset and the total mass distribution, it is significant that equation (5) still gives a good parameterization of the anisotropy in redshift-space power on scales as small as 1000 km/s.

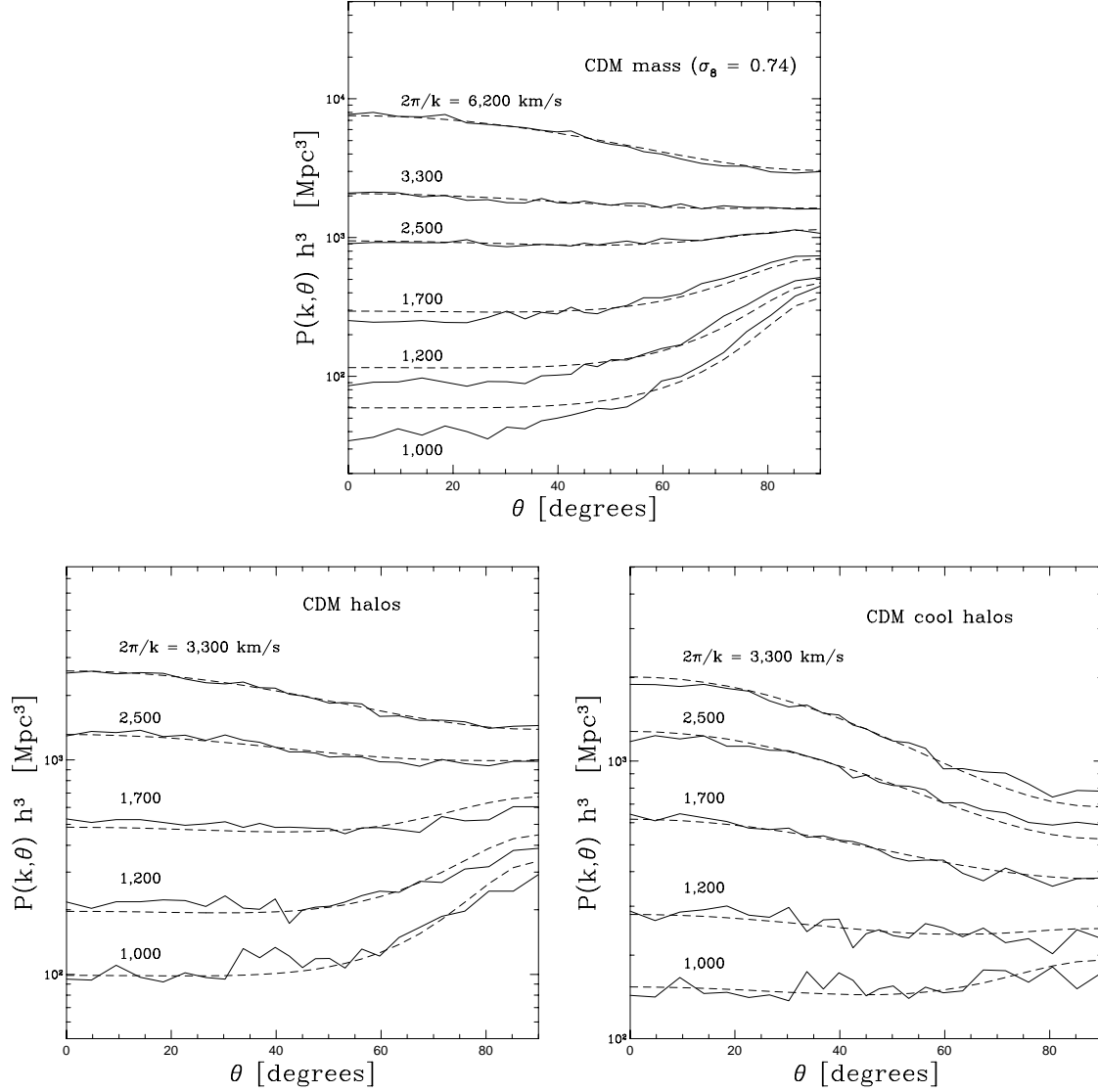


Fig. 1.—The two-dimensional redshift-space power spectrum. The power is shown as a function of the angle between the observer’s line of sight and the wavevector for several different wave magnitudes  $k$ . The plots show the power for (a) the mass distribution, (b) the galaxy halos, and (c) the culled halo subset with low small-scale velocity dispersion. The solid lines show the estimate of  $P$  from the simulation data, while the dashed curves are the predicted power using the multiplicative filter model in equation (5). A  $\beta$  value of unity was used to generate the model curves in all three plots, while the  $\sigma_v$  values were 1,100 km/s, 850 km/s, and 400 km/s, for the mass, halos, and cool halos, respectively.

### 3 Dispersion-Corrected Sampling Functions

With a working model for the anisotropy of redshift-space power in the presence of linear flows and small-scale dispersion, we now focus on measuring  $\beta$  from the linear contribution. The strategy is to determine the small-scale  $\sigma_v$  in a redshift-space density field, remove the effects of the dispersion signal as modeled by equation (3), and then to estimate the anisotropy in Fourier modes on scales  $\gtrsim 10 h^{-1}$  Mpc. In an infinite or periodic dataset one could simply remove the dispersion signal by a mathematically trivial deconvolution. For example, the redshift-space distribution might be transformed into the Fourier domain and the result multiplied by the inverse square-root of  $F_D$ . However, in practice the finite size of a survey complicates the calculation of the Fourier transform especially at large wavelengths (e.g., Fisher et al. 1993). Here we circumvent the problem of the Fourier transform and deconvolution of the density field by making use of sampling functions which are tuned to be orthogonal to the dispersion signal. We consider only the variances in such samples and hence we extract linear flow information without directly deconvolving the redshift-space density field.

A sampling function  $s$  gives a sample value  $S$  at some randomly chosen origin by projection onto a density field  $\rho$

$$S = \int d^3x s(\mathbf{x})\rho(\mathbf{x})/\bar{\rho} , \quad (6)$$

where  $\bar{\rho}$  is the mean density. It is straightforward to show that the variance in a set of many samples taken at random locations in a density field is related to the power spectrum  $P$  by

$$\sigma^2 = \int d^3k P(\mathbf{k})|s(\mathbf{k})|^2 . \quad (7)$$

Of course in practical applications the power is inferred from a discrete galaxy distribution which contains shot noise. In this case, equation (7) must be corrected by subtracting the shot noise contribution,

$$\frac{1}{\bar{n}} \int d^3k |s(\mathbf{k})|^2 , \quad (8)$$

where  $\bar{n}$  is the mean density of galaxies.

Anticipating the case when  $P$  in equation (7) is set to  $P_s$ , we choose  $s$  to have a form such that in the Fourier domain

$$s(\mathbf{k}) = \frac{s_L(k, \mu)}{\sqrt{F_D(k, \mu)}} \quad (9)$$

where  $s_L$  couples only to the signal from linear flows. We assume that the samples are taken at a large distance from the observer so that  $\mu$  is constant. Clearly, the effect of the denominator in equation (9) is to perform the deconvolution of dispersion signal at the level of random sampling.

We emphasize that in the method discussed here we evaluate only sample variances; we never directly perform a deconvolution of the redshift-space density field to remove shot noise and the effects of small-scale dispersion. Yet the filter given in equation (9) might be used to do just that. Indeed, deconvolution with this filter and the inverse of the linear flow filter in equation (4) would enable the real-space density field to be reconstructed from redshift-space data even on linear scales which are affected by small-scale dispersion. This would advance known reconstruction techniques (e.g., Yahil et al. 1994; Nusser & Davis 1995; Tegmark & Bromley 1996) by introducing some correction for nonlinear dispersion. However, the filtering must be done with great attention to noise and finite boundary effects. For example, optimal (Wiener) filtering is necessary to avoid the pitfalls of deconvolution in the presence of noise (cf. Press et al. 1992, section 13.3).

With the nonlinear effects removed, the remaining problem is to determine a measure of  $\beta$  from the linear flow signal in the sample variances. Bromley (1994) proposed an estimator that follows from taking the derivative of some radially symmetric function  $R$  along the direction of a unit vector  $\mathbf{n}$  at fixed orientation relative to the distant observer:

$$s_L(\mathbf{x}) = \mathbf{n} \cdot \nabla R(x) \quad \text{and} \quad s_L(\mathbf{k}) = i\mathbf{n} \cdot \mathbf{k} R(k) . \quad (10)$$

Here  $R$  is specified to have the form of a 1D Gaussian,

$$R(x) = \exp(-x^2/2\lambda^2) , \quad (11)$$

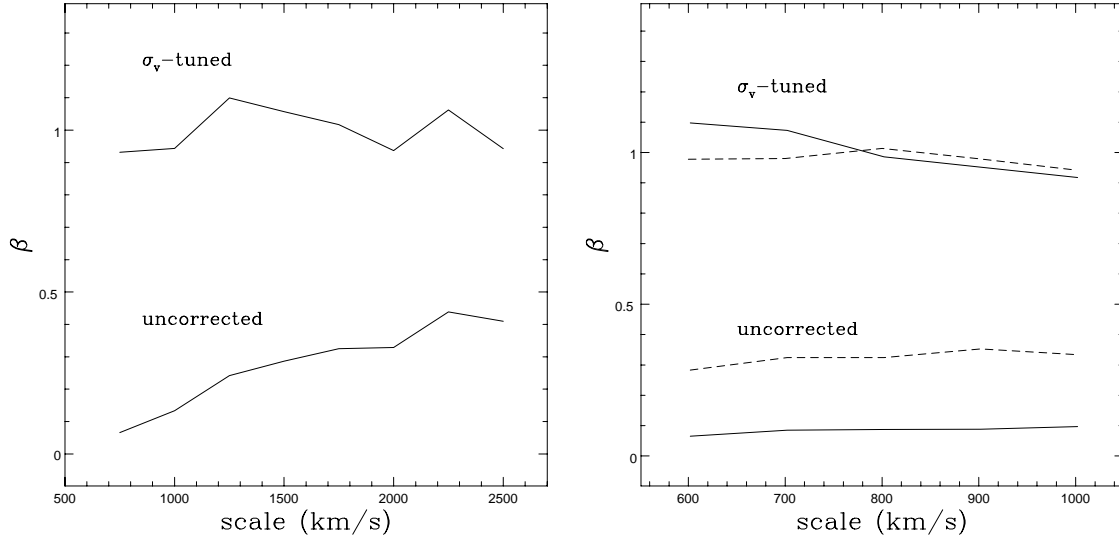


Fig. 2.—The recovered  $\beta$  from the ratio-of-sample-variances method. In the plots the upper curves were obtained using a  $\sigma_v$  of 1,000 km/s, 850 km/s, and 425 km/s, respectively, and the lower curves have no correction for dispersion ( $\sigma_v = 0$ ). The results for the mass distribution are shown in (a) while (b) contains the results for the full halo catalog (solid curves) and the culled halo subset (dashed curves).

where  $\lambda$  defines the scale of the sampling function. The effects of linear flows in redshift space can be isolated by considering two different sampling functions constructed from equations (9) through (11), with  $\mathbf{n}$  directed parallel and perpendicular to the observer's line of sight, respectively. The ratio of variances  $\sigma_{\parallel}$  and  $\sigma_{\perp}$  from these two sampling functions is easily related to  $\beta$  using equation (7) and the redshift-space power in equation (5):

$$\mathcal{Q}(f) \equiv \frac{\sigma_{\parallel}^2}{\sigma_{\perp}^2} = \frac{1 + \frac{6}{5}\beta + \frac{3}{7}\beta^2}{1 + \frac{2}{5}\beta + \frac{3}{35}\beta^2}. \quad (12)$$

The desired measure of  $\beta$  comes from inverting this expression. Note that the result does not require any knowledge the form of the real-space power spectrum as all dependence on  $P_R$  cancels out in the ratio  $\mathcal{Q}$ .

We use the numerical data described in §2 to determine how well the ratio of sample variances in equation (12) is able to recover the correct value of  $\beta$ . Figure 2 illustrates the recovery from redshift space data over a range of sample function scales  $\lambda$  using known values of the velocity dispersion  $\sigma_v$  on megaparsec scales from the three simulation catalogs. The method works well for the mass particles and the full halo catalog; in both cases the estimate of  $\beta$  was within 10% of its true value. In the culled halo subset the recovered  $\beta$  was consistently about 10% below the expected  $\beta$  value of  $\sim 1.1$  when the  $\sigma_v$  value of  $\sim 384$  km/s at  $1 h^{-1}$  Mpc pair separation was used in the sample functions. However, the number of halo pairs at a megaparsec is separation is relatively small, and with  $\sigma_v$  itself rising fairly rapidly with separation at this scale, it is not surprising that a higher value of  $\sigma_v$ , 450 km/s, gives a better estimate of  $\beta$ .

Figure 2 shows strong promise for determination of  $\beta$  with the ratio-of-variances method using tuned sample functions. The figure also illustrates the importance of taking into account small-scale velocity dispersion. Even on redshift-space scales approaching three times the pairwise velocity dispersion, we find that the recovered  $\beta$  is generally more than 50% below its true value.

## 4 The IRAS 1.2 Jy Survey

We now apply the sampling method to the IRAS 1.2Jy redshift catalog. This is a flux-limited sample of 5,665 galaxies with sky coverage of 87.6% (Strauss et al. 1990; Yahil et al. 1991). The flux limit in the  $60\mu\text{m}$  waveband causes the density of galaxies in redshift space to fall off rapidly with distance; the number of galaxies in radial bins peaks at  $\sim 5,000$  km/s, and the selection function  $\phi$  which describes the fall-off has

a value of less than 0.01 at 10,000 km/s. We take this latter distance to be the characteristic size of this survey.

The sampling procedure used to generate  $\mathcal{Q}$  from the IRAS galaxies is more complicated than in the case of the pristine numerical data. First, each galaxy must be weighted by  $1/\phi$  to account for low-luminosity neighbors which were missed because of the flux limit. Here we use a selection function of the form given by Yahil et al. (1991, eq [11] therein) with  $a = 0.51$ ,  $b = 1.84$ , and  $hr_* = 5,440$  km/s.

The second complication in the analysis of the IRAS galaxies is that shot noise is significant on scales below  $\sim 1,000$  km/s. If uncorrected it causes the sampling method to underestimate  $\mathcal{Q}$  and  $\beta$ . Here, we remove the shot noise contribution to the sample variance by equation (8); its amplitude for each level of sampling scale is determined from artificial catalogs generated by a Monte Carlo method in accordance with the selection function of the IRAS galaxies.

A third complication is incomplete sky coverage. So that we may sample with the tuned functions without the limitations of avoiding unobserved sky regions, we follow the strategy of Fisher, Scharf & Lahav (1994) who interpolated the IRAS galaxy density field across the zone of avoidance and other unsampled regions. The interpolation method is described in detail by Yahil et al. (1991) as part of a broader algorithm to reconstruct the real-space density field from the IRAS data in redshift space. Here, we include 705 artificial galaxies in real space kindly provided by M. Davis. Since the artificial galaxies represent a real-space density field, reconstructed under the assumption that  $\beta$  is unity, the line-of-sight mode amplitudes are guaranteed to be smaller than in the redshift-space IRAS catalog. This effect, along with the fact that any discontinuity with a boundary running along the observer's line of sight will only add power to wavemodes in the plane of the sky, can drive the ratio  $\mathcal{Q}$  to systematically lower values, causing  $\beta$  to be underestimated.

There is one final difference between the analyses of the numerical data and of real galaxies. In the simulations the pairwise velocity dispersion  $\sigma_v$  at megaparsec scales, the essential ingredient of the dispersion filter (eq. [3]), is known precisely from the 6D phase space distribution of particles or halos. In contrast,  $\sigma_v$  must be gleaned from redshift-space data in the IRAS catalog, a procedure which introduces some uncertainty into  $\sigma_v$ . Published values (e.g., Fisher et al. 1993) and our own estimate based on the method of Davis & Peebles (1983) suggest that  $\sigma_v$  for the IRAS 1.2Jy sample is 320 km/s with  $1-\sigma$  errors of  $\sim 40$  km/s. If the IRAS galaxies are unbiased tracers of mass, then the numerical data suggest that this value should be appropriate for use in the dispersion filter (eq. [3]). If the IRAS population is antibiased with respect to mass, as has been conjectured on the basis of comparison with optical surveys, then the use of this value may generate an underestimate of  $\beta$ . This conclusion is made on the basis of similarity between our culled halo catalog and the IRAS galaxies which both have relatively low number densities in cluster cores (as compared to the full halo catalog or optically selected galaxies) and rapidly rising  $\sigma_v$  values on megaparsec scales.

After implementing the above modifications to the sampling procedure, we analyzed the IRAS galaxies which lie inside a thick spherical shell of redshift space with an inner radius of 500 km/s and an outer radius chosen between 10,000–12,000 km/s. We estimate uncertainty by determining an approximate number of spatially independent samples that can be measured with a given sampling function and make repeated measurements of  $\beta$  with that number of samples. In Figure 3 we show the recovered  $\beta$  from the IRAS galaxies as a function of sampling scale. In general, the recovered value has a lower bound at the  $1-\sigma$  level which is above 0.4. The mean of the recovered  $\beta$  on scales of 600–1,500 km/s is in the range of 0.6–0.9, depending on the outer radius of the sampled region. This sensitivity shows up mostly at the larger sampling scales where we must balance a desire to use as big a volume as possible against the increasing sparsity of galaxies at large distances. The most stable estimate comes from sampling scales around 1,000 km/s for which we find that  $\beta = 0.8^{+0.4}_{-0.3}$  at the  $1-\sigma$  level. As noted above, the systematic error from zone-of-avoidance interpolation is likely to cause this value of  $\beta$  to be a slight underestimate of the true value. The similarity between the IRAS galaxies and our culled halo catalog in §3 mentioned above suggests that our choice of  $\sigma_v = 320$  km/s may be too small, leading to a more significant underestimate of  $\beta$ . Indeed,  $\sigma_v$  in the IRAS data climbs to a value of 370 km/s at  $2 h^{-1}$  Mpc which leads to a  $\sim 10\%$  increase in  $\beta$  over the estimates shown in Figure 3.

## 5 Conclusions

Here we have modeled the anisotropy in the redshift-space power spectrum on the basis of Peacock & Dodds' (1994) theoretical work and our own high-resolution numerical simulations. We demonstrated that the effects of nonlinear clustering on the power spectrum of both mass and galaxy halos can be described

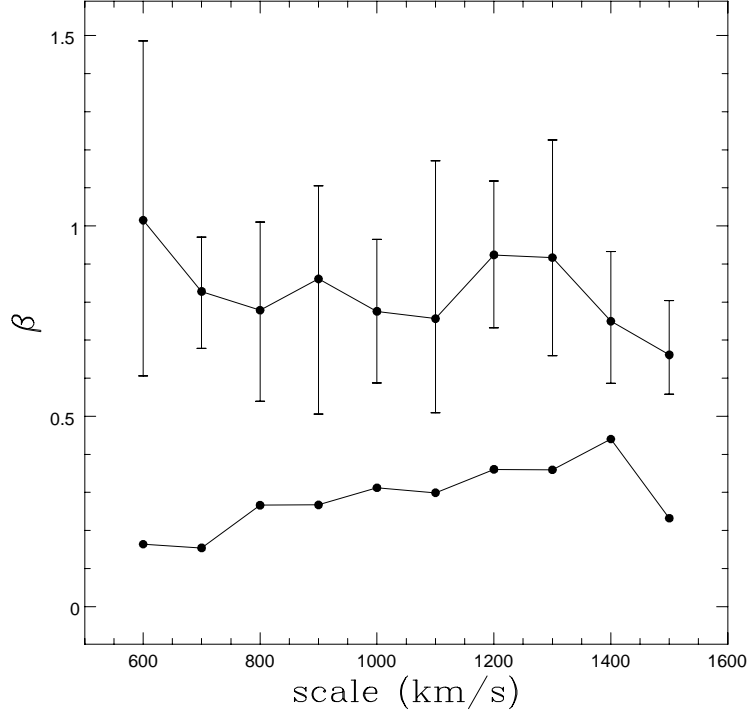


Fig. 3.—The recovered  $\beta$  from the IRAS 1.2Jy survey. The upper curve comes from sampling with functions tuned to  $\sigma_v = 320$  kms, and the lower curve corresponds to zero dispersion. The error bars show the  $1\text{-}\sigma$  uncertainties.

by a simple multiplicative filter function. The filter-based model for the anisotropy is seen to hold on linear and translinear scales which are greater than the pairwise velocity dispersion  $\sigma_v$  at megaparsec. Having established the domain of validity, we then proposed a redshift-space statistic, the ratio of sample variances in equation (12), which is both impervious to the effects of nonlinear clustering and sensitive to linear flows and the linear growth parameter  $\beta$ .

This work should serve as a cautionary reminder of the extent to which the nonlinear distortions contaminate signal from linear flows (e.g., Fig. 2). In a measurement of power, the magnitude of contamination can be determined from equation (3); for example the nonlinear signal reduces the power along the line of sight by  $\sim 40\%$  at scales  $2\pi/k$  equal to five times  $\sigma_v$ . In the case of the IRAS 1.2Jy survey where  $\sigma_v$  is  $\sim 300$  km/s, the presence of nonlinear distortion is important even at scales of 1,500 km/s. For optical surveys where  $\sigma_v \sim 800$  km/s (e.g., Marzke et al. 1995), the extent of contamination exceeds 4,000 km/s.

The work presented here also gives the encouraging message that nonlinear clustering does not prevent the accurate extraction of  $\beta$  from linear flows in redshift data. Figure 2 provides the evidence based upon cosmological simulations of the greatest dynamical range published to date.

The analysis of the IRAS 1.2Jy survey is intended to show that the method introduced here gives reasonable results. Our estimate of  $\beta = 0.8^{+0.4}_{-0.3}$  is consistent at the  $2\text{-}\sigma$  level with virtually every published value based on an analysis of the IRAS galaxies (e.g., Dekel et al. 1993; Fisher, Scharf & Lahav 1994; Cole, Fisher & Weinberg 1995; Hamilton 1995), although it is higher than most redshift-space measurements based exclusively on linear theory. Our procedure is most similar in spirit to that of Hamilton (1995) who obtained  $\beta = 0.69^{+0.21}_{-0.19}$  using a merged IRAS 1.2Jy and the optically selected QDOT catalog. Hamilton, working with harmonics of a smoothed power spectrum, modeled the nonlinear noise in the same way as we did, with an isotropic exponential distribution for pairwise velocities. We hope that we have provided convincing evidence in §2 that such a model is indeed robust.

We note that our estimate of  $\beta$  from the IRAS 1.2Jy sample is consistent with dynamical analyses that make use of real-space information as well as redshift data. For example, Dekel et al. (1993) find  $\beta = 1.28 \pm 0.3$  on scales of 1,500–4,000 km/s. While knowledge of both real-space and redshift distributions can yield a more accurate measurement of  $\beta$ , the real-space data are difficult to measure and therein lies the value in



extracting  $\beta$  directly from redshift space. Fisher et al. (1994) performed a pure redshift-space analysis of the IRAS 1.2Jy galaxies by modeling  $\xi_s$  and obtained  $\beta = 0.45^{+0.27}_{-0.18}$  on scales of 1,000–1,500 km/s, well below the dynamical estimates at larger scales. However, the modeling of  $\xi_s$  and its dependence on the uncertain behavior of the velocity distribution function is difficult. In contrast, our measure of  $\beta$ , based instead on a much simpler model in the Fourier domain, is consistent with the large-scale dynamical results.

The ultimate hope is that our measure may be applied to forthcoming optical surveys which exhibit high  $\sigma_v$  values (e.g., Marzke et al. 1995). With the ability to accurately determine  $\beta$  down to translinear scales in spite of nonlinear clustering effects, our measure can provide high statistical significance in a correlation analysis of redshift-space data.

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#### REFERENCES

- Brainerd, T. G., Bromley, B. C., Warren, M. S., & Zurek, W. H. 1996, *ApJ*, 464, L103  
 Bromley, B. C. 1994, *ApJ*, 423, L81  
 Cole, S., Fisher, K. B., Weinberg, D. H. 1995, *MNRAS*, 275, 515  
 Davis, M. & Peebles, P. J. E. 1983, *ApJ*, 267, 465  
 Dekel, A., Bertschinger, E., Yahil, A., Strauss, M. A., Davis, M., & Huchra, J. P. 1993, *ApJ*, 412, 1  
 Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A., & Huchra, J. P. 1993, *ApJ*, 402, 42  
 Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A., & Huchra, J. P. 1994, *MNRAS*, 267, 927  
 Fisher, K. B., & Nusser, A. 1996, *MNRAS*, 279, L1  
 Fisher, K. B., Scharf, C. A., & Lahav, O. 1994, *MNRAS*, 266, 219  
 Gramann, M., Cen, R. & Bahcall, N. A. 1993, *ApJ*, 419, 440  
 Hamilton, A. J. S. 1992, *ApJ*, 385, L5  
 Hamilton, A. J. S. 1993, *ApJ*, 406, L47  
 Hamilton, A. J. S. 1995, preprint (astro-ph/9507022)  
 Kaiser, N. 1987, *MNRAS*, 231, 149  
 Marzke, R. O., Geller, M. J., da Costa, L. N., & Huchra, J. P. 1995, *AJ*, 110, 477  
 Nusser, A., & Davis, M. 1994, *ApJ*, 421, L1  
 Peacock, J. A., & Dodds, S. J. 1994, *MNRAS*, 267, 1020  
 Peebles, P. J. E. 1980, *The Large-Scale Structure of the Universe* (Princeton: Princeton University Press).  
 Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, *Numerical Recipes in C* (Cambridge: Cambridge University Press).  
 Strauss, M. A., Davis, M., Yahil, A., & Huchra, J. P. 1990, *ApJ*, 361, 49  
 Tadros, H., & Efstathiou, G. 1996, *MNRAS*, in press (astro-ph/9603016)  
 Tegmark, M. & Bromley, B. C. 1995, *ApJ*, 453, 533  
 Warren, M. S. 1995, in *WWW Proceedings of the Heron Island Workshop on Peculiar Velocities*, ed. P. J. Quinn (<http://qso.lanl.gov/~heron/Warren/warren.html>)  
 Yahil, A., Strauss, M. A., Davis, M., & Huchra, J. P. 1991, *ApJ*, 372, 380  
 Zurek, W. H., Quinn, P. J., Salmon, J. K., & Warren, M. S. 1994, *ApJ*, 431, 559